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LETTER TO THE EDITOR

The ac magnetic response in type-II superconductors

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Abstract. The ac response of type-II superconductors to an alternating magnetic field is numerically studied on the basis of the time-dependent Ginzburg–Landau equations. We examine the temperature dependence of the ac susceptibility associated with a small ac magnetic field in the absence of a bias dc field. It is shown that with increasing temperature the in-phase component of the fundamental susceptibility exhibits a step-like change from a negative constant value to zero, while the out-of-phase component of the fundamental susceptibility and the third-harmonic component have a peak at a certain temperature near the superconducting transition temperature. These results are in qualitative agreement with those of recent experiments on high-T_c superconductors.

Magnetization measurements using alternating fields have been widely employed in the study of type-II superconductors [1]. This is because in these experiments the effective time window can be easily changed by varying the frequency of the applied ac field. Thus, the study of the ac magnetic response of type-II superconductors provides direct information on flux dynamics in these materials. Recently, several authors have measured the ac susceptibility in high-T_c materials as functions of various physical parameters such as temperature, and the frequency and amplitude of the applied ac field [2–5].

Various models have been proposed to explain experimental data for the ac susceptibility of type-II superconductors, and especially of high-T_c materials [6–12]. Although these models have partially succeeded in explaining the ac magnetic response of type-II superconductors, they are at rather macroscopic and/or phenomenological levels, and thus are still incomplete. In this letter, to complement the previous study, we attack the above problem by using a different type of approach, that is, the numerical approach of the time-dependent Ginzburg–Landau (TDGL) equations [13, 14]. The main advantage offered by this computer simulation study is the ability to visualize the dynamical processes of magnetization and thus to directly obtain information on the dynamics of the magnetic flux structure without making any of the *ad hoc* electrodynamical assumptions used in previous models (e.g., the field dependence of the critical current in the critical state model). Here, performing numerical calculation of the TDGL equations, we examine the ac magnetic response of type-II superconductors to a small alternating magnetic field in the absence of the steady bias field. In particular, we discuss the temperature dependence of the ac susceptibility with the amplitude and frequency of the ac field being fixed.

The TDGL equations are composed of two partial differential ones for the complex order parameter $\psi(\boldsymbol{r}, t)$ and the vector potential $\boldsymbol{A}(\boldsymbol{r}, t)$ at time t and position $\boldsymbol{r}$ [13–15]:

$$\frac{\hbar^2}{2mD}\left(\frac{\partial}{\partial t} + \mathrm{i}\frac{e\phi}{\hbar}\right)\psi = -\frac{\delta F}{\delta \psi^*} + f(\boldsymbol{r}, t) \qquad (1)$$

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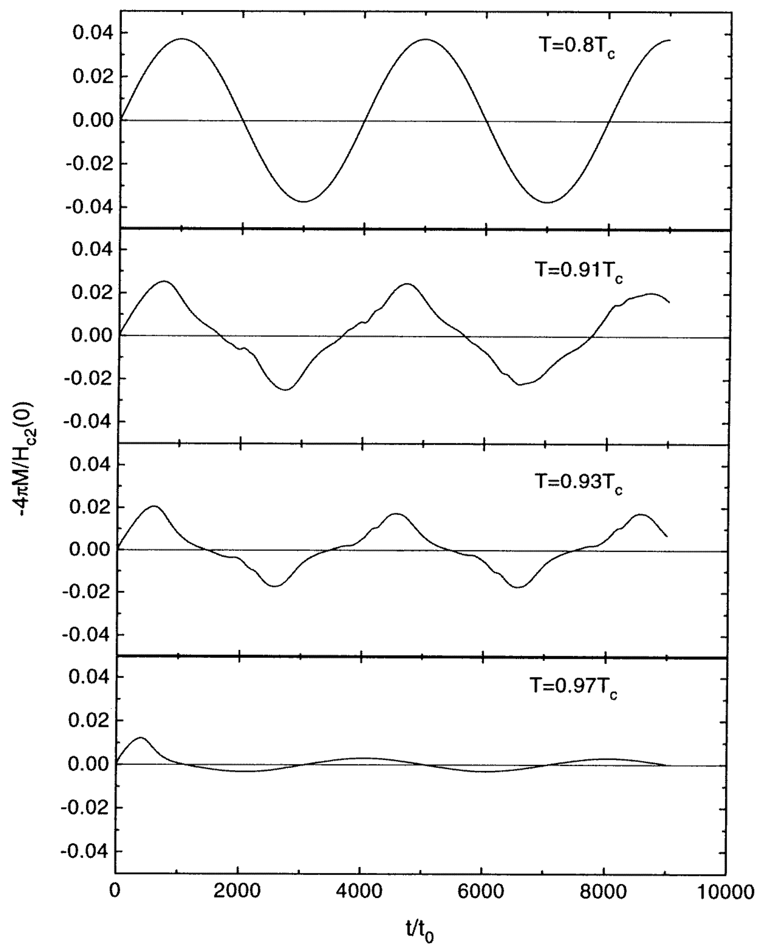

Figure 1. Time variation of the magnetization $M(t)$ for $T/T_c = 0.8$, 0.91, 0.93, and 0.97.

$$\sigma\left(\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{\nabla}\phi\right) = -\frac{\delta F}{\delta \boldsymbol{A}} \tag{2}$$

with the covariant time derivative $(\partial/\partial t + \mathrm{i}e\phi/\hbar)$ and a scalar potential ϕ, where ψ^* denotes the complex conjugate of ψ. These equations are invariant under the local U(1) gauge transformation for ψ, $\boldsymbol{A}$, and ϕ. Here, D and σ are the diffusion constant and the conductivity, associated with the normal phase, respectively, and they have the relation [16]

$$\sigma = \frac{c^2\xi^2}{48\pi\lambda^2}\frac{1}{D} \tag{3}$$

with the coherence length ξ and the magnetic penetration depth λ. The last term of the r.h.s. of equation (1), $f(\boldsymbol{r}, t)$, denotes the thermal noise with zero mean, i.e. $\langle\langle f\rangle\rangle = 0$, and the correlation

$$\langle\langle f^*(\boldsymbol{r}', t')f(\boldsymbol{r}, t)\rangle\rangle = 12\xi_0^{-4}t_0k_BT\left(\frac{H_c(0)^2}{8\pi}\right)^{-1}\delta(\boldsymbol{r}' - \boldsymbol{r})\delta(t' - t) \tag{4}$$

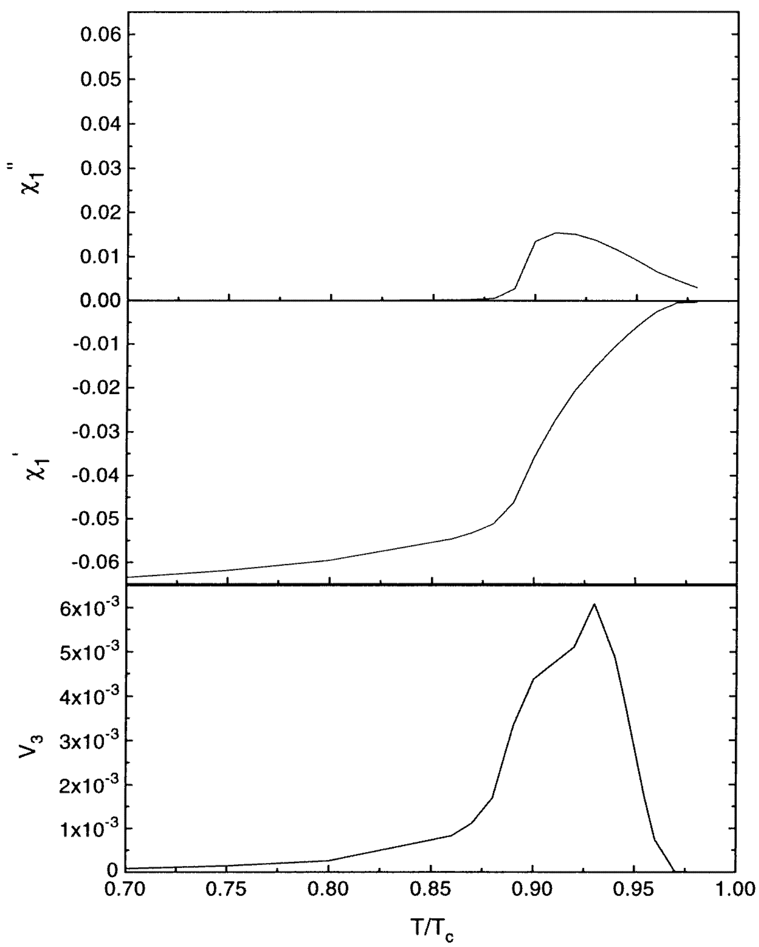

Figure 2. Temperature dependences of χ_1', χ_1'', and V_3.

where $\langle\langle\cdots\rangle\rangle$ denotes the ensemble average, ξ_0 is the coherence length at zero temperature, and $t_0 \equiv \pi\hbar/(96k_BT_c)$ with the superconducting transition temperature T_c at zero field. The Ginzburg–Landau (GL) free-energy functional $F[\psi, \boldsymbol{A}]$ is given by

$$F = \int \mathrm{d}\boldsymbol{r}\ \left[\frac{1}{2m}|\boldsymbol{D}\psi|^2 + \alpha(T)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{8\pi}(\boldsymbol{\nabla}\times\boldsymbol{A})^2\right] \tag{5}$$

with the covariant derivative $\boldsymbol{D} \equiv -\mathrm{i}\boldsymbol{\nabla} - (e/c)\boldsymbol{A}$ and the local magnetic flux density $\boldsymbol{b}(\boldsymbol{r}, t) \equiv \boldsymbol{\nabla}\times\boldsymbol{A}$. Under the assumption that $\alpha(T) = \alpha(0)(T/T_c - 1)$, and $\alpha(0)$ and β are positive constants in equation (5), the upper and lower critical fields are given by $H_{c2}(T)/H_{c2}(0) = 1 - T/T_c$ and $H_{c1}(T)/H_{c2}(0) = (\ln\kappa/(2\kappa^2))(1 - T/T_c)$, respectively, with the GL parameter κ. The other notation is conventional [16]. These equations are supplemented with boundary conditions, $\boldsymbol{D}\psi|_n = 0$ and $\boldsymbol{\nabla}\times\boldsymbol{A}|_s = \boldsymbol{H}_e$, where the index n denotes the normal direction on the sample boundary and the index s denotes the sample boundary with an applied magnetic field $\boldsymbol{H}_e$.

We here consider a type-II superconductor in the x–y plane with a size $L_x \times L_y$. The

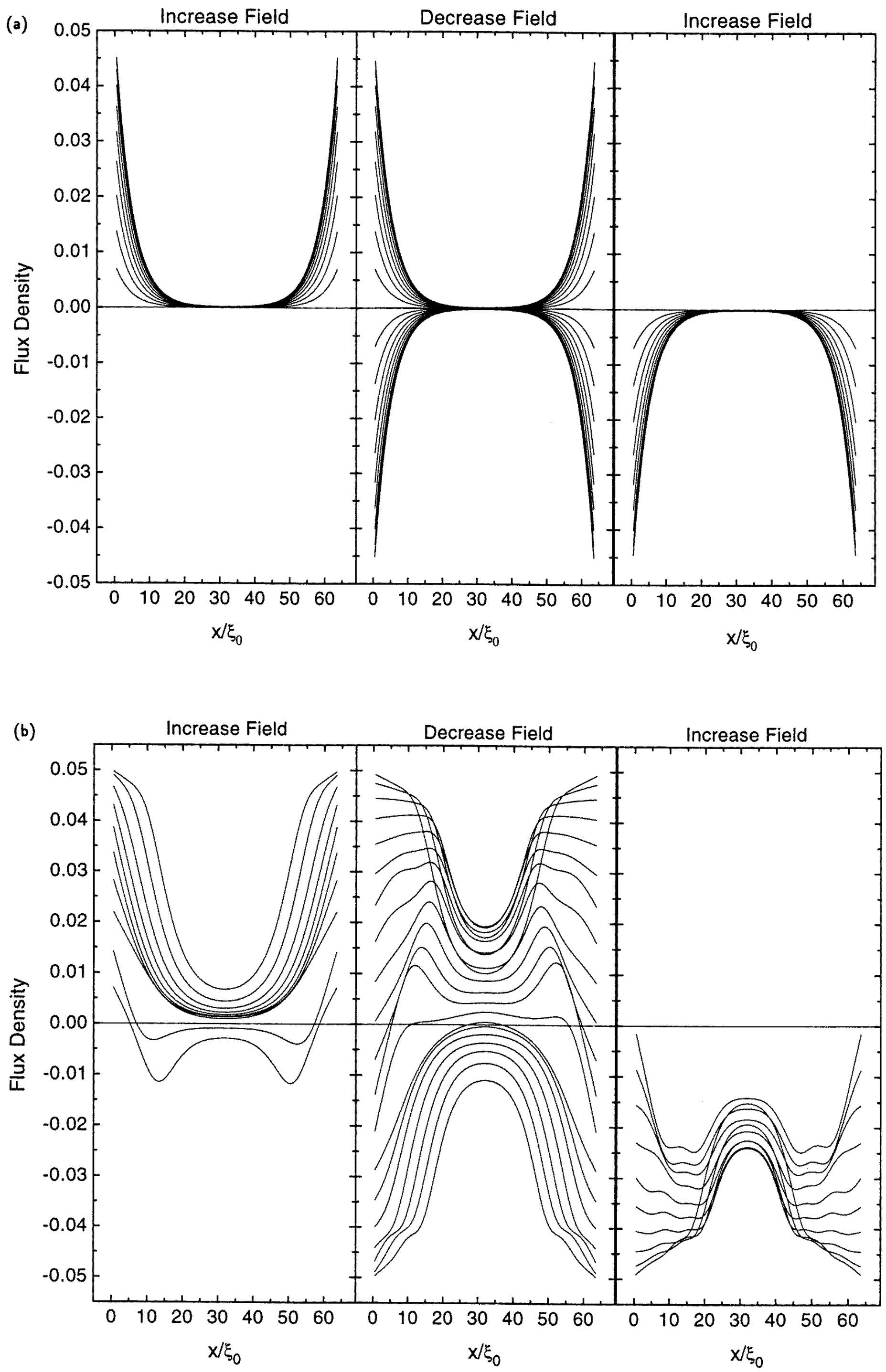

Figure 3. See facing page.

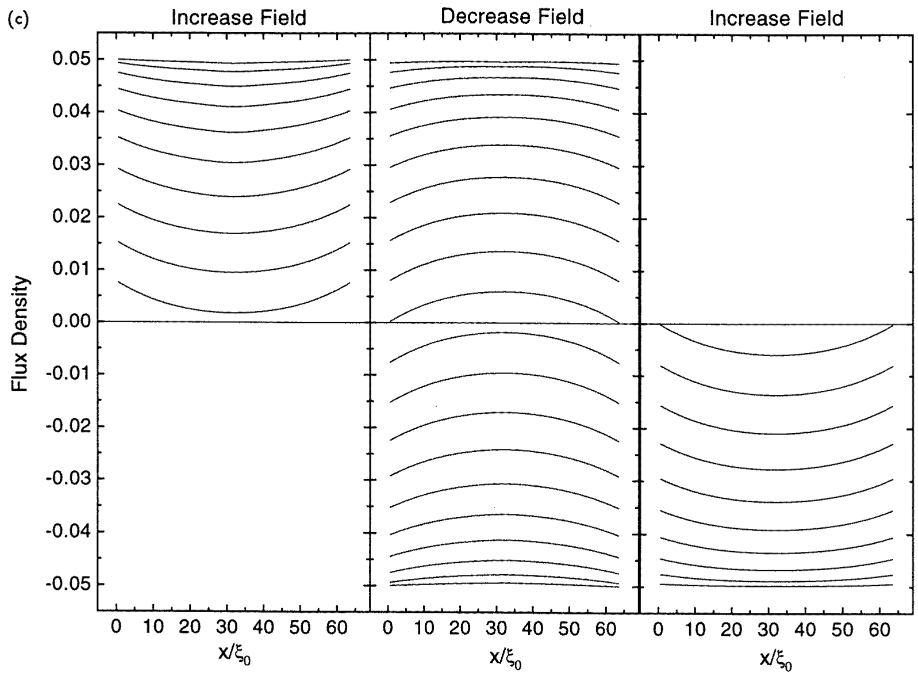

Figure 3. (Continued) Time variations of profiles of the z-component of the local magnetic flux density, b_z, in units of $H_{c2}(0)$ along the x-axis at $y = 32\xi_0$ for $T/T_c = 0.8$ (a), 0.91 (b), and 0.97 (c). Time goes from the left-hand column to the right-hand one for one cycle of the ac field.

sample is assumed to be infinite in the z-direction, and the problem is reduced to two dimensions neglecting all derivatives along z. The external ac magnetic field is applied to the sample as $\boldsymbol{H}_e = H(t)\hat{\boldsymbol{z}}$ with $H(t) = H_{ac}\sin\omega t$ where H_{ac} and ω denote the amplitude and angular frequency of the ac field, respectively, and $\hat{\boldsymbol{z}}$ is the unit vector along the z-axis. In actual simulations the TDGL equations are transformed into the dimensionless discretized equations on a two-dimensional lattice by introducing link variables for the vector potential and the gauge fixing such that the scalar potential is set to zero. Since these procedures are the same as those in references [13, 14], we will not discuss the numerical procedures any further.

In the following simulations, we set $L_x = L_y = 64\xi_0$ and $\kappa = 2$. The amplitude and frequency of the ac field are fixed to be $H_{ac} = 0.05H_{c2}(0)$ and $\omega t_0/2\pi = 0.25 \times 10^{-3}$ (that is, the period of the ac field is $4000t_0$), respectively. We also take the lattice spacing and time step for numerical calculations to be $0.5\xi_0$ and $0.0125t_0$, respectively. These values are chosen for the computational reason of obtaining efficient results within our computer availability. As the initial state we choose the zero-field-cooling state.

In figure 1 the magnetization, $M(t)$, of the sample is plotted against time for various values of temperature. The magnetization $M(t)$ is defined as $4\pi M(t) = \langle B\rangle(t) - H(t)$, where the magnetic induction $\langle B\rangle(t)$ is obtained from the sample average of the z-component, $b_z(\boldsymbol{r}, t)$, of the local magnetic flux density $\boldsymbol{b}(\boldsymbol{r}, t)$. At $T = 0.8T_c$, the magnetization is sinusoidal in nature, according to the external ac field $H_{ac}\sin\omega t$. With increasing temperature, the magnetization deviates from the sinusoidal character with a decrease of its magnitude, and simultaneously a phase-shift phenomenon occurs in the t–$M(t)$ curve. Further increase of temperature results in the large degree of phase shift. Indeed, at $T = 0.97T_c$, the magnetization becomes almost zero at times when the applied ac field is maximum (minimum). Note that even for no bulk pinning case, the phase-shift phenomena take place due to the sample boundary effect, as has been discussed in our previous work [17].

To give details of the ac magnetic response of the system, we study the ac susceptibility defined by the Fourier transformation of the magnetization $M(t)$:

$$M(t) = H_{ac} \sum_{n=1}^{\infty} (\chi_n' \sin n\omega t - \chi_n'' \cos n\omega t) \tag{6}$$

where χ_n' and χ_n'' $(n = 1, 2, \ldots)$ denote the in-phase and out-of-phase components of the nth-harmonic susceptibility, respectively, with the nth harmonics $V_n \equiv \sqrt{\chi_n'^2 + \chi_n''^2}$. In calculating the ac susceptibility, simulation data during the first period have been discarded to avoid transient effects. In the present case we have numerically checked that only odd harmonics are generated [8]. In figure 2, χ_1', χ_1'', and V_3 are plotted as functions of temperature; they have been often measured experimentally. It is shown that with increasing temperature, χ_1' exhibits a step-like change from a negative constant value $(=-1/(4\pi)$ theoretically [6]) to zero, while χ_1'' initially rises from zero, goes through a maximum at $0.91T_c$ (called the peak temperature, denoted by T_p), and then returns to a small value near T_c. Note that $H_{c1}(T_p) \simeq 0.008 < H_{ac} = 0.05 < H_{c2}(T_p) = 0.09$ for $T_p = 0.91T_c$ in units of $H_{c2}(0)$. The third harmonic V_3 is also found to have a similar temperature dependence to χ_1'' with the peak temperature $T_p = 0.93T_c$. These results are qualitatively consistent with those of recent experiments on high-T_c superconductors [2–5]. At present it is unclear whether the slight difference between the peak temperature estimated from χ_1'' and that from V_3 is physically meaningful or not.

Now we relate the above macroscopic behaviour of the system to the spatio-temporal structure of the local magnetic flux. In figures 3(a)–3(c) the time evolution of the profile of the local magnetic flux density b_z along the x-axis is shown at $y = 32\xi_0$ for a complete cycle of the ac field for $T/T_c = 0.8$ (a), 0.91 (b), and 0.97 (c), respectively. In figure 4 the time evolution of the profile of the y-component of the current density is shown along the x-axis at $y = 32\xi_0$ for $T = 0.91T_c$ during one half of the period of the ac field, as well as b_z. The current density is given by $(c/4\pi)\nabla \times \nabla \times \boldsymbol{A}$ in units of $j_0 \equiv cH_{c2}(0)/(4\pi\xi_0)$. No definite magnetic vortex structures are observed in figures 3(a)–3(c). It is also found that the step in χ_1' is due to the transition from near-perfect screening (figure 3(a)) to complete penetration $(\langle B\rangle(t) \simeq H(t))$ of the ac field impinging into the whole sample (figure 3(c)). Moreover, we have numerically checked that the smallest temperature at which penetrating magnetic flux (current) reaches to the centre of the sample is $0.90T_c$ in the present case. Thus, the peak in χ_1'' (and maybe the peak in V_3) corresponds to the first penetration of the flux (current) to the centre of the sample (figure 3(b) and figure 4). Although such an interpretation has been already proposed by several authors [7, 9, 10], this is the first simulation study to discuss the ac magnetic response of type-II superconductors from the point of view of the local magnetic flux dynamics without making any of the *ad hoc* electrodynamical assumptions used in previous models. Finally, we remark that the magnetic relaxation phenomenon can be seen in figure 3(b) and figure 4. Indeed, even when the external field changes to a decreasing stage from the initial ramp-up phase, the magnetic flux near the centre of the sample still increases for a while. An importance of this relaxation behaviour has recently been pointed out as one of possible causes for the frequency dependence of the ac susceptibility [3, 12]. Such magnetic relaxation effects on the ac susceptibility will be

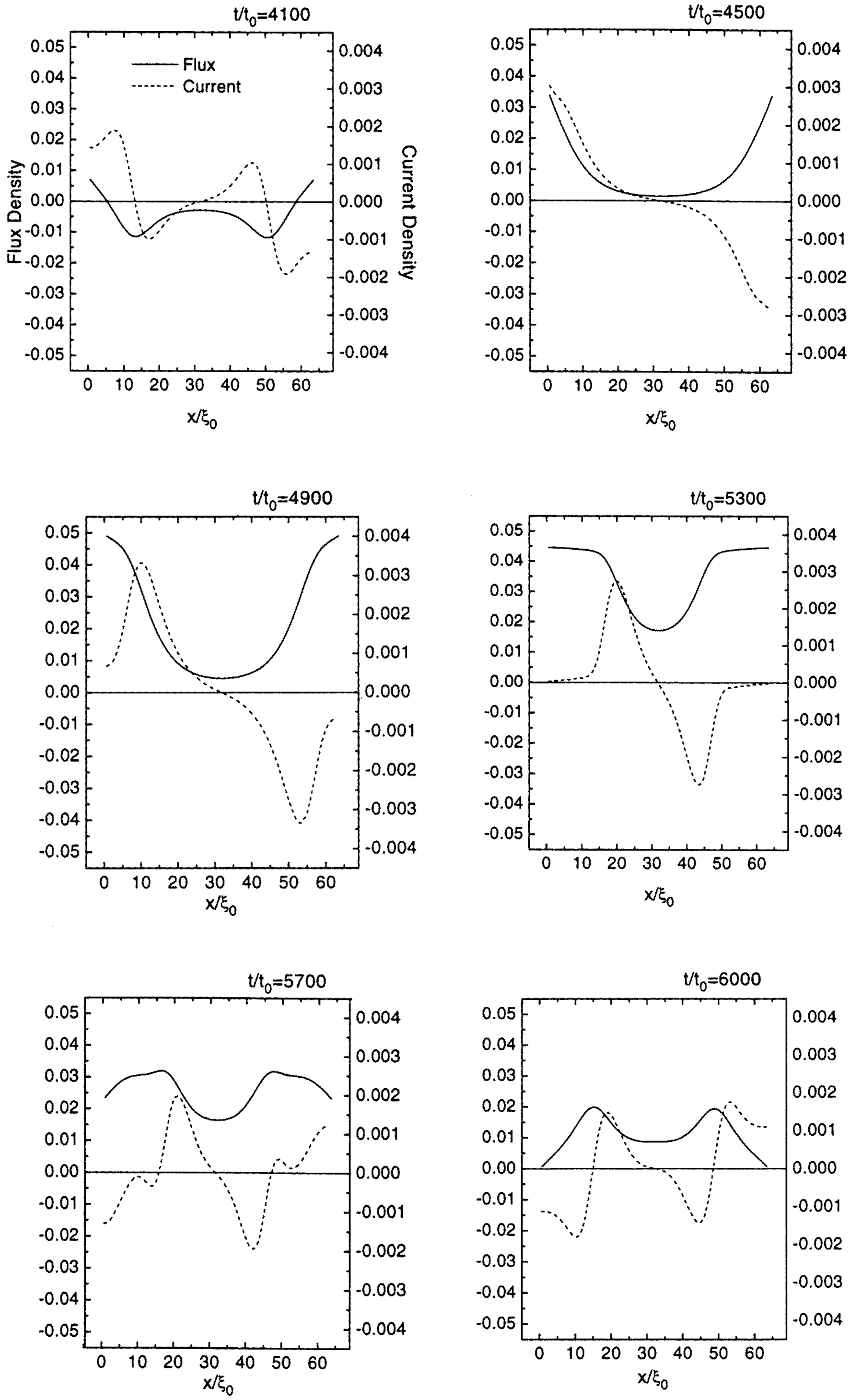

Figure 4. Time variations of profiles of the y-component of the current density (dashed lines) in units of $j_0 = cH_{c2}(0)/(4\pi\xi_0)$ along the x-axis at $y = 32\xi_0$ for $T/T_c = 0.91$ during one half of the period of the ac field. Profiles of b_z are also shown for comparison, as solid lines.

discussed elsewhere.

In conclusion, we have studied the ac magnetic response of type-II superconductors to the alternating magnetic field on the basis of numerical calculation of the TDGL equations. In particular, the temperature dependence of the ac susceptibility in the absence of the dc field with the amplitude and frequency of the ac field being fixed has been discussed. We have found a step-like change in χ_1', and a peak in χ_1'' and also in the third harmonic V_3 at a certain temperature near T_c. These results are in qualitative agreement with recent experimental data on high-T_c superconductors. Moreover, we have briefly discussed the relationship between these macroscopic behaviours of the system and the spatio-temporal behaviour of the local magnetic flux.

However, we should mention that there can be a big distance between simulations and experiments. This is because there are the following faults in the present model. Firstly, in real systems the ac response is strongly affected by pinning and thermal fluctuations, which are not in the model. Secondly, we have neglected the z-dependence of the problem. The present results, therefore, apply only to films, not bulk materials where the entanglement of flux lines may be important. These points will be discussed in future work.

Nevertheless, the present approach based on computer simulations of TDGL equations has been found to be potentially rich for throwing novel light on the problem studied here. For instance, the present model is applicable for discussing the validity of assumptions used in macroscopic and/or phenomenological models. Since the present study is still in a primitive stage, detailed simulations are now under way to allow discussion of a quantitative comparison of simulation results with theoretical results and experimental data obtained by not only the ac method but also the Hall probe method [18]. Moreover, many interesting problems still remain open, such as the dependence of the ac response on various physical parameters (e.g., the amplitude and frequency of the ac field, the dc field, bulk pinning and the sample geometry), and also the universal behaviour of the ac susceptibility. These problems are also now under consideration.

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References

[1] For a review, see
Hein R A, Francavilla T L and Liebenberg D H (ed) 1991 *Magnetic Susceptibility of Superconductors and Other Spin Systems* (New York: Plenum)
[2] Ghatak S K, Mitra A and Sen D 1992 *Phys. Rev.* B **45** 951
[3] Wolfus Y, Abulafia Y, Klein L, Larkin V A, Shaulov A, Yeshurun Y, Konczykowski M and Feigel'man M 1994 *Physica* C **224** 213
[4] Ding S Y, Wang G Q, Yao X X, Peng H T, Peng Q Y and Zhou S H 1995 *Phys. Rev.* B **51** 9107
[5] Kumaraswamy B V, Lal R and Narlikar A V 1996 *Phys. Rev.* B **53** 6759
[6] Bean C P 1964 *Rev. Mod. Phys.* **36** 31
[7] Chen D X, Nogues J and Rao K V 1989 *Cryogenics* **29** 800
[8] Ji L, Sohn R H, Spalding G C, Lobb C J and Tinkham M 1989 *Phys. Rev.* B **40** 10 936
[9] Geshkenbein V B, Vinokur V M and Fehrenbaher R 1991 *Phys. Rev.* B **43** 3748
[10] van der Beek C J, Geshkenbein V B and Vinokur V M 1993 *Phys. Rev.* B **48** 3393
[11] Shatz S, Shaulov A and Yeshurun Y 1993 *Phys. Rev.* B **48** 13 871
[12] Prozorov R, Shaulov A, Wolfus Y and Yeshurun Y 1995 *Phys. Rev.* B **52** 12 541
[13] For a review, see
Enomoto Y, Kato R and Maekawa S 1993 *Studies of High Temperature Superconductors* vol 11, ed A V Narlikar (New York: Nova Science) p 309
[14] Fram H, Ullah S and Dorsey A T 1991 *Phys. Rev. Lett.* **66** 3067
Liu F, Mondello M and Goldenfeld N 1991 *Phys. Rev. Lett.* **66** 3071

Kato R, Enomoto Y and Maekawa S 1991 *Phys. Rev.* B **44** 6916; 1993 *Phys. Rev.* B **47** 8016
[15] Schmid A 1969 *Phys. Rev.* **180** 527
[16] Tinkham M 1975 *Introduction to Superconductivity* (New York: McGraw-Hill)
[17] Enomoto Y, Ishikawa Y and Maekawa S 1996 *Physica* C **263** 21
[18] Zeldov E, Majer D, Konczykowski M, Larkin A I, Vinokur V M, Geshkenbein V B, Chikumoto N and Shtrikman H 1995 *Europhys. Lett.* **30** 367